

doc-
u-
ment

2 This
page
is
here
to
make
the
page
num-
bers
come
out
cor-
rectly.

Do
not
print
this
page.

My

The-

sis

Ti-

tle

A
The-
sis
Pre-
sented
to
The

Fac-
ulty
of
the
Math-
e-
mat-
ics
Pro-
gram
California
State
Uni-
ver-
sity
Chan-
nel
Is-
lands

In
(Par-
tial)
Ful-
fill-
ment
of
the
Re-
quire-
ments
for
the
De-
gree

Masters
of
Sci-
ence

by

Shawn
Richard-
son

April,
2014

©
2014
Shawn
Richard-
son
ALL
RIGHTS

RE-
SERVED

Signature
page
for
the
Mas-
ters
in
Math-
e-
mat-
ics
The-
sis
of

APPROVED FOR THE MATHEMATICS PROGRAM

~~Thesis Advisor's name, Thesis Advisor~~ ~~Date~~

~~Committee Member's name, Thesis Committee~~ ~~Date~~

APPROVED FOR THE UNIVERSITY

~~Dr. Gary A. Berg, AVP Extended University~~ ~~Date~~

To
my
par-
ents,
name
and
name,
in
grat-
i-
tude
for
their
en-
cour-
age-
ment
and
sup-
port.

Acknowledgements

Abstract

My
The-
sis
Ti-
tle
by
Shawn
Richard-
son

Type
in
your
ab-
stract
here.

CONTENTS

1.	
Introduction	1
2.	
Related	
Work	1
3.	
Methodology	1
3.1.	
Initial	
Hypothesis	1
4.	
Results	2
5.	
Discussion	
and	
Future	
Work	2
5.1.	
Notation	
and	
Terminology	3
6.	
Hypothesis	4
6.1.	
Initial	
Hypothesis	
Results	4

6.2.	
Revision	
1	4
6.3.	
Revision	
2	4
7.	
Data	
Used	
for	
Analysis	5
7.1.	
FVC2002:	
Fingerprint	
Verification	
Competition	5
8.	
Minutiae	
Extraction	6
8.1.	
Image	
Preparation	6
8.2.	
Skeletonization	6
8.3.	
Identify	
Minutiae	8
8.4.	
Minutiae	
Properties	8

9.	
Analysis	8
9.1.	
Triangles	8
9.2.	
Principal	
Component	
Analysis	8
9.3.	
Matching	8
10.	
Stuff	
to	
Include	8
10.1.	
Fingerprint	
Matching	8
10.2.	
Data	
Collected	8
10.3.	
Triangulation	9
10.4.	
Training	
Data	11
10.5.	
Principle	
Component	
Analysis	11

1. INTRODUCTION

In
an
at-
tempt
to
match
fin-
ger-
prints
sam-
pled
at
dif-
fer-
ent
res-
o-
lu-
tions
and
taken
at
dif-
fer-
ent
an-
gles,
tri-
an-
gles
were
gen-
er-
ated
from
the

minu-
tia
points.
These
tri-
an-
gles
were
ini-
tially
com-
pared
based
on
the
an-
gles
be-
tween
the
points
with-
out
re-
gard
to
the
length
of
the
sides.
This
was
done
to
ab-
stract
the
fin-
ger-
prints

from
the
res-
o-
lu-
tion
of
the
orig-
i-
nal
im-
age
as
well
as
the
an-
gle
at
which
the
print
was
scanned.

2. RELATED

WORK

3. METHODOLOGY

3.1. Initial

Hy-
poth-

e-
sis.

Af-
ter
ex-
tract-
ing
the
co-
or-
di-
nates
of
the
minu-
tia
points,
a
tri-
an-
gle
can
be
gen-
er-
ated
for
each
unique
set
of
3
minu-
tia.
Each
of
these

tri-
an-
gles
can
be
stored
in
a
data-
base
as
a
pair
of
num-
bers,
one
for
each
of
the
 2
largest
an-
gles.
The
third
an-
gle
is
not
re-
quired
as
the
sum
of
the
an-
gles
must

equal
180
de-
grees.
By
stor-
ing
only
the
two
an-
gles,
res-
o-
lu-
tion,
ori-
en-
ta-
tion,
and
in-
ver-
sion
of
the
im-
age
are
ab-
stracted.
Fin-
ger-
prints
could
then
be
com-
pared
by
the

num-
ber
of
tri-
an-
gles
in
each
im-
age
that
match.

4. RESULTS

800
fin-
ger-
prints
in
data-
base.
17639
end
points
and
13015
bi-
fur-
ca-
tion
points.

5. DISCUSSION

AND

FU-

TURE

WORK

5.1. Notation
and
Ter-
mi-
nol-
ogy.
(As
needed...)

6. HYPOTHESIS

6.1. Initial

Hy-
poth-
e-
sis
Re-
sults.

Us-
ing
the
[In-
sert
Data-
base
Name]
fin-
ger-
print
data-
base,
ex-
per-
i-
men-
tal
re-
sults
show
that
fin-
ger-
print
im-
ages
with
more
minu-
tia
scored

sig-
nif-
i-
cantly
higher
than
those
with
fewer
minu-
tia
points,
even
if
the
im-
ages
were
not
of
the
same
fin-
ger-
print.
[Add
test
re-
sult
data
and
method-
ol-
ogy]

6.2. Revision

1.
In
or-
der
to

re-
duce
false
pos-
i-
tives,
sev-
eral
re-
vi-
sions
were
made.
The
length
of
the
longest
side
was
added.

6.3. Revision

2.

Us-
ing
the
largest
an-
gle
as
a
start-
ing
point,
the
next
an-
gle
in
a

clock-
wise
ro-
ta-
tion
was
se-
lected,
re-
gard-
less
of
size.
The
di-
rec-
tion
of
the
minu-
tia
was
then
added
to
the
data
set
for
each
of
the
3
minu-
tia
points.

7. DATA

USED

FOR

ANAL-

Y-

SIS

7.1. FVC2002:

**Fin-
ger-
print
Ver-
i-
fi-
ca-
tion
Com-
pe-
ti-
tion.**

Of
the
four
databased
made
avail-
able
for
the
2002
Fin-
ger-
print

Ver-
i-
fi-
ca-
tion
Com-
pe-
ti-
tion,
the
im-
ages
in
set
A
of
DB1
were
cho-
sen
for
this
anal-
y-
sis.
The
im-
ages
in
this
data-
base
were
op-
ti-
cally
scanned
us-
ing
an

Iden-
tix
TouchView
II.
The
re-
sult-
ing
im-
ages
are
388
x
374
pix-
els
with
a
res-
o-
lu-
tion
of
500
dots
per
inch.
A
to-
tal
of
90
vol-
un-
teers
with
an
av-
er-
age
age

of
20,
were
used
to
gen-
er-
ate
the
im-
ages
with
30
of
the
90
se-
lected
at
ran-
dom
used
for
DB1.
The
fore-
fin-
ger
and
mid-
dle
fin-
ger
of
each
hand
was
sam-
pled
for
each

of
the
30
vol-
un-
teers.
A
to-
tal
of
12
sam-
ples
of
each
fin-
ger
were
tak-
ing
dur-
ing
three
dis-
tinct
ses-
sions
with
at
least
two
weeks
sep-
a-
rat-
ing
each
ses-
sion.
Ef-
forts

were
made
to
in-
crease
fin-
ger
place-
ment,
dis-
place-
ment,
ro-
ta-
tion
(not
to
ex-
ceed
35
de-
grees),
and
mois-
ture
di-
ver-
sity.
The
sen-
sors
were
not
sys-
tem-
at-
i-
cally
cleaned,
nor
were

ef-
forts
made
to
con-
trol
the
qual-
ity
of
the
im-
ages.

?,

]]maio_{vc}2002 :?

???

Of
the
120
fin-
gers,
the
im-
ages
of
the
10
high-
est
qual-
ity
fin-
gers
were
dis-
carded.

Of
the
re-
main-
ing

110
fin-
gers,
100
were
des-
ig-
nated
as
set
A
and
10
as
set
B,
with
8
of
the
12
sam-
pled
im-
pres-
sions
in-
cluded
?,
]]maio_fvc2002 :?
???.Myresearchutilizedthe800imagesmadeavailableassetAofDB1.

8. MINUTIAE

EX-

TRAC-

TION

8.1. Image Prepa-

ra-
tion.

The
im-
ages
used
are
op-
ti-
cal
scans
of
fin-
gers
at
500
dots
per
inch
in
8
bit
gray
scale
ash
shown
in
Fig-
ure
??.

Utilizing

the
DIP-
im-
age
tool-
box
within
MAT-
LAB,
the
im-
ages
were
en-
hanced
to
re-
duce
mis-
clas-
si-
fi-
ca-
tion
of
ridge
end-
ings
and
bifrica-
tion
points.
Each
fin-
ger-
print
im-
age
was
di-
vided

into
blocks
of
 $16 \times$
16
pix-
els
for
which
the
vari-
ance
of
each
block
was
com-
puted.
The
vari-
ance
was
then
com-
pared
with
the
thresh-
old
value.
If
the
vari-
ance
of
a
block
was
less
than
the

thresh-
old
value,
then
the
block
was
deleted
from
the
orig-
i-
nal
fig-
ure.
This
pro-
cess
was
car-
ried
out
for
the
en-
tire
im-
age.
The
re-
sult-
ing
im-
age
was
then
nor-
mal-
ized
to
get

the
de-
sired
vari-
ance
of
the
given
im-
age.
The
blocks
were
then
fil-
tered
along
the
di-
rec-
tion
of
the
ori-
en-
ta-
tion
an-
gle
us-
ing
the
value
of
the
fre-
quency
ob-
tained
for
each

block.
A
bi-
nary
mask
was
used
to
re-
move
points
around
the
edges
as
well
as
in-
ter-
nal
points
with
a
low
re-
li-
a-
bil-
ity
in-
dex
as
gen-
er-
ated
by
the
im-
age

en-
hance-
ment
pro-
cess.
Fil-
ter-
ing
was
done
for
each
block
us-
ing
the
lo-
cal
ori-
en-
ta-
tion
an-
gle
and
fre-
quency.

8.2. Skeletonization.

A
bi-
nary
im-
age
was
gen-
er-
ated
from
the

age

pre-
pared
im-
age
as
shown
in
Fig-
ure
??.
The
bi-
nary
im-
age
was
then
skele-
tonized
by
re-
mov-
ing
the
outer
layer

of
pix-
els
un-
til
a
sin-
gle
pixel
wide
line
re-
mained.
End
pix-
els
were
then
iden-
ti-
fied
as
those
pix-
els
with
ex-
actly
pixel
within
the
8
pos-
si-
ble
sur-
round-
ing
pix-
els.
A

mask
was
ap-
plied
to
these
re-
sults
to
elim-
i-
nate
the
ar-
eas
around
the
edge
which
would
oth-
er-
wise
gen-
er-
ate
false
end
points,
as
well
as
ar-
eas
of
low
con-
fi-
dence
where
the

age

im-
age
was
not
suf-
fi-
ciently
clear.

8.3. Identify Minu- tiae.

Each
of
the
end
points
re-
main-
ing
af-
ter
the
mask-
ing
pro-
cess

was
de-
ter-
mined
to
be
an
end
point.

8.4. Minutiae Prop- er- ties.

9. ANALYSIS

9.1. Triangles.

9.2. Principal Com- po- nent Anal- y- sis.

9.3. Matching.

10. STUFF

TO

IN-

CLUDE

10.1. **Fingerprint Match-**

ing.

Re-

li-

ably

match-

ing

im-

ages

of

fin-

ger-

prints

is

ex-

tremely

dif-

fi-

cult,

mainly

due

to

large

vari-

a-

tions

in

im-

ages

of

the

same

fin-
ger.
The
main
fac-
tors
re-
spon-
si-
ble
for
these
large
intra-
class
vari-
a-
tions
are:
dis-
place-
ment,
ro-
ta-
tion,
par-
tial
over-
lap,
non-
linear
dis-
tor-
tion,
vari-
able
pres-
sure,
chang-
ing
skin

con-
di-
tions,
noise,
and
fea-
ture
ex-
trac-
tion
errors?,
]31]maltoni_h*andbook*₂003.

10.2. Data Col- lected.

For
each
im-
age
pro-
cessed,
the
minu-
tia
points
are
de-
ter-
mined.
Then
for
each
minu-
tia
point,
the
lo-
ca-
tion,

direction,
and type
are determined.
The location
is extracted
as the xy -coordinate
in the image
and the direction
is the angle
relative to
a line
running

from
the
ori-
gin,
in-
creas-
ing
along
the
 x
axis.
The
type
is
clas-
si-
fied
as
ei-
ther
an
end
or
bi-
fur-
ca-
tion
point.

10.3. **Triangulation.**

The
minu-
tia
are
then
grouped
into
sets
of
3,

form-
ing
a
tri-
an-
gle.
Us-
ing
the
 xy -
coordinates
of
each
point,
the
an-
gles
and
lengths
of
each
side
can
be
cal-
cu-
lated.
For
each
tri-
an-
gle,
the
largest
an-
gle
was
first
de-
ter-
mined,

then
tak-
ing
a
clock-
wise
ro-
ta-
tion,
the
sec-
ond
an-
gle
is
de-
ter-
mined.
Along
with
the
dis-
tance
be-
tween
minu-
tia
at
the
ve-
rac-
i-
ties
of
these
two
an-
gles,
the

tri-
an-
gle
can
be
be
fully
rep-
re-
sented.
Hypnosis:
Us-
ing
the
cen-
ter
point
of
the
tri-
an-
gle
as
a
ref-
er-
ence
would
min-
i-
mize
the
im-
pact
of
er-
rors
caused
ei-
ther
by

dis-
tor-
tion
or
er-
rors
in
the
ex-
trac-
tion
pro-
cess.
The
di-
rec-
tion
of
the
minu-
tia
point
is
then
cal-
cu-
lated
rel-
a-
tive
to
the
di-
rec-
tion
to
the
cen-
ter
of
the

tri-
an-
gle.
This
point
was
cho-
sen
to
re-
duce
the
im-
pact
of
er-
rors
in
the
lo-
ca-
tions
of
the
minu-
tia,
ei-
ther
by
dis-
tor-
tion
or
er-
rors
in
the
ex-
trac-
tion

process.

Proof.

Let

A, B

and

C

be

ar-

bi-

trary

minu-

tia

points.

Let

a

be

the

dis-

tance

be-

tween

points

B

and

C ,

then

$a =$

$$\sqrt{(B_x - C_x)^2 + (B_y - C_y)^2}$$

$b =$

$$\sqrt{(A_x - C_x)^2 + (A_y - C_y)^2}$$

$c =$

$$\sqrt{(A_x - B_x)^2 + (A_y - B_y)^2}$$

□

$\theta =$

$\arctan(\frac{\textit{opposite}}{\textit{adjacent}}) =$

$\arctan(\frac{\Delta x}{\Delta y})$

$\cot \theta =$

$(\frac{\textit{adjacent}}{\textit{opposite}}) =$

$(\frac{\Delta y}{\Delta x})$

If

$\Delta y <$

0,

$\theta =$

$$\theta + \pi$$

If

$$\Delta x = 0, \theta = \frac{\pi}{2}$$

10.4. Training

Data.

The data-base used consist of 8 different images of 100 unique fingers for a total of 800 fingerprint images. One image of finger was chosen,

using
the
random
function
within
MySQL,
and
reserved
for
the
testing
set.
The
remaining
700
images
were
used
as
a
training
set.
Statistic
for
the
training
data
are
shown
in

Table
??.
Angle
 α ,
being
the
largest
angle
of
a
tri-
an-
gle
was
ex-
pected
to
have
a
min-
i-
mum
value
of
60
de-
grees
and
a
max-
i-
mum
value
of
less
than
180

de-
grees.
The
val-
ues
shown
for
 α
in
Ta-
ble
??
when
trans-
lated
from
ra-
di-
ans
to
de-
grees
show
the
sam-
pled
min-
i-
mum
value
to
be
ap-
prox-
i-
mately
60.0685
de-
grees
and
the

sam-
pled
max-
i-
mum
value
of
ap-
prox-
i-
mately
179.9667
de-
grees.
An-
gle
 β
be-
ing
ar-
bi-
trar-
ily
one
of
the
re-
main-
ing
an-
gles
is
ex-
pected
to
be
greater
than
0
de-
grees

and
less
than
90
de-
grees.
The
ex-
per-
i-
men-
tal
data
con-
firms
this
with
re-
sults
of
ap-
prox-
i-
mately
0.0064
and
88.5559
de-
grees.
The
re-
main-
ing
com-
po-
nents
are
sim-
i-
larly

Component	Minimum	Maximum	Average	σ
Angle α	1.0483928950	3.1410122706	1.7797521977	0.4721377805
Angle β	0.0001109755	1.5455915815	0.6718969221	0.3484298449
Direction a	-3.1415926535	3.1415926536	0.0167706757	1.8339501956
Direction b	-3.1415926535	3.1415926536	-0.0314165278	1.8484983038
Direction c	-3.1415926535	3.1411944932	-0.0277245946	1.8392294858
Length	2.2360679775	335.6262802583	58.0791589147	31.5340026405

Dataset

con-
sis-
tent
with
their
ex-
pected
val-
ues.

10.5. Principle

Com-
po-
nent
Anal-

y-
sis.

Af-
ter
cal-
cu-
lat-
ing
the
sta-
tis-
ti-
cal

in-
for-
ma-
tion
for
the
lin-
ear
com-
po-
nent
for
the
tri-
an-
gles,
each
com-
po-
nent
was
nor-
mal-
ized
by
sub-
tract-
ing
the
av-
er-
age,
then
di-
vid-
ing
by
the
stan-
dard

de-
vi-
a-
tion
following
data:
(data
types).
Prin-
ci-
ple
com-
po-
nent
anal-
y-
sis
was
then
used
to
re-
duce
the
num-
ber
of
cri-
te-
ria
to
be
com-
pared.