Rational Distances to the Corners of the Unit Square

By

Victor M. Moreno

A DISSERTATION SUBMITTED IN PARTIAL FULFILLMENT OF THE

REQUIREMENTS FOR THE DEGREE OF

MASTERS IN SCIENCE

(MATHEMATICS)

at the

CALIFORNIA STATE UNIVERSITY - CHANNEL ISLANDS

2012

(You really need to replace this single-one-page-file Here are the)

APPROVAL INSTRUCTIONS

1. Go and open the file named: ApprovalBeforeScanning.tex
2. Make the appropriate substitutions in the file (instructions are also found inside the file).
3. Compile that file with Latex, then dvitops, then pspdf
4. Print it
5. Sign it, Scan it, Save it as Approval.pdf in this folder
6. When you save it, the computer will ask you if you want to replace it with a file that already exists, say yes!

Dedication

I dedicate this work to all the women who have loved me, specially to Shakira.

ACKNOWLEDGEMENTS

This was done with the aid and help of Dr. Garcia, and Dr. Ivona Grze from California State University Channel Islands.

Camarillo, California

June 14, 2012

ABSTRACT

The aim of this talk is to find a concrete statement about the existence of Rational Points in the Unit Square. We will use the following definition for a rational point: A rational point is defined as a point whose distances to the vertices of a geometric figure are all rational. Notice that a number theoretic Rational Point is a point (x, y) in an algebraic curve $f(x, y) = 0$ where x and y are rational. We will show a variety of methods to prove certain properties of these rational points. Finally we prove an equivalence between non-existence of rational points on the edges of the unit square and the absence of integer roots for certain families of polynomials.

Table of Contents

CHAPTER 1

INTRODUCTION

The idea of rationality is an old concept in Mathematics. This topic gained strength during the times of Greek mathematician Pythagoras; he believed that everything could be expressed as a ratio of two integers, the irony is that it was his theorem $a^2 + b^2 = c^2$ (the Pythagorean theorem) that led other mathematicians to the existence of irrational numbers. This was an embarrassment to him and his followers that it was believed that some of the early Mathematicians that proved the existence of irrational numbers (by proving $\sqrt{2}$ is not rational) were killed. Many mathematicians don't believe that the pythagoreans were capable of killing one of their own because he made a great discovery. On the other hand, the pythagoreans did kill at least one of their members

(Hippasus) because he divulge the pythagorean secret of how to inscribe a dodecahedron inside a sphere. If they did it once what would prevent them from doing it again? If the pythagoreans were a group of mathematicians with the goal of discovering mathematics then why did they stop after proving $\sqrt{2}$ was irrational? Theodorus show all the irrational numbers up to $\sqrt{17}$ and then he just stop and finally Theaetetus generalized Theodorus work (See [**?**]).

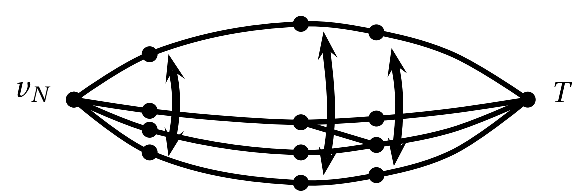


Figure 1. Vertices along an arrow belong to the same equivalence class.

Once the irrational numbers were established, then the Reals and Complex completed our number system. All of the sudden we are flooded with all this numbers with little or no properties, and at the same time we have all this objects in which the concept of rationality must be proven. One of the first questions emerging was does there exists geometric figures with rational sides and rational areas? Heron created a general formula to create right triangles with rational sides. In 1921 Professor L.E Dickson showed that all shapes of rational points could be created by the juxtaposition of two right triangles (see [**?**]). In 1929 W. Fitch Cheney used the rational triangles to find Heronian triangles, where all the sides and the area are integers (see [**?**])

Definition 1.0.1 *A rational point is a point* (x, y) *whose distances to the vertices of a geometric figure are all rational (see figure 1).*

Figure 1.1: A picture of a gull.

In chapter 4 we explore the idea of a rational point on the edges of the unit square. We use Pythagorean triples at first since we are trying to generalize the notion of these rational points on the edges of the unit square. By using the results from Guy and an a polynomial function generated from the generalization of the existence of rational points on the edges of the square we concluded that these polynomial functions have no integer roots, being the following the main result on this thesis.

Figure 1.2: A picture of the same gull looking the other way!

Theorem 1.0.2 *There are no p, q, r positive integers satisfying $p^2 - q^2 > 2pq$ such that $f(p,q) = p^4 - 2p^3q + 2pq^3 + q^4 - 2r^2 = 0$*

Finally, we were able to create a series of corollaries that were easily deduced from the last theorem.

1	2	3
4	5	6
7	8	9

Table 1.1: A simple table

Notice how the tables and figures have independent counters.

Here blah blah blah. Then of course more blah blah blah. Finally yeah yeah yeah. Here blah blah blah. Then of course more blah blah blah. Finally yeah yeah yeah. Here blah blah blah. Then of course more blah blah blah.

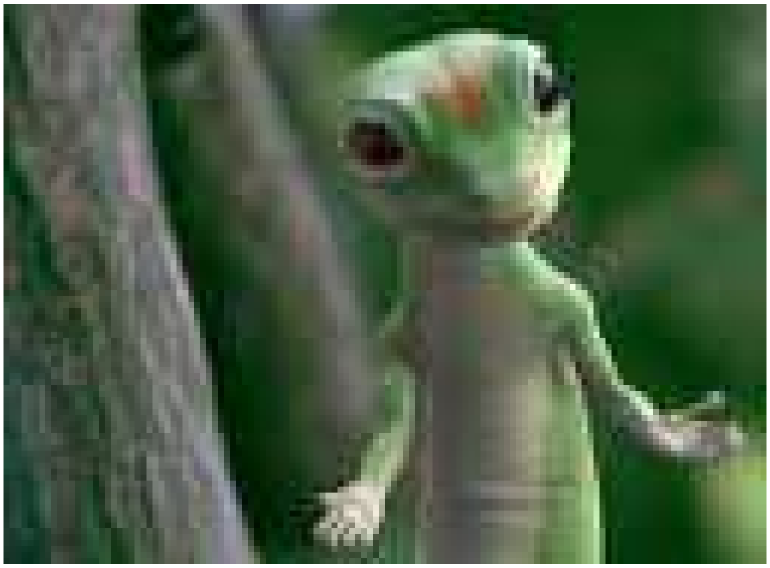


Figure 1.3: Close up of *Hemidactylus* sp., which is part the genus of the gecko family. It is the second most speciose genus in the family.

Finally yeah yeah yeah.Here blah blah blah. Then of course more blah blah blah. Finally yeah yeah yeah. Here blah blah blah. Then of course more blah blah blah. Finally yeah yeah yeah.

Here blah blah blah. Then of course more blah blah blah. Finally yeah yeah yeah. Here blah blah blah. Then of course more blah blah blah. Finally yeah yeah yeah. Here blah blah blah. Then of course more blah blah blah. Finally yeah yeah yeah.Here blah blah blah. Then of course more blah blah blah. Finally yeah yeah yeah. Here blah blah blah. Then of course more blah blah blah. Finally yeah yeah yeah. Here blah blah blah. Then of course more blah blah blah. Finally yeah yeah yeah.

Here blah blah blah. Then of course more blah blah blah. Finally yeah yeah yeah. Here blah blah blah. Then of course more blah blah blah. Finally yeah yeah yeah. Here blah blah blah. Then of course more blah blah blah. Finally yeah yeah yeah.Here blah blah blah. Then of course more blah blah blah. Finally yeah yeah yeah. Here blah blah blah. Then of course more blah blah blah.

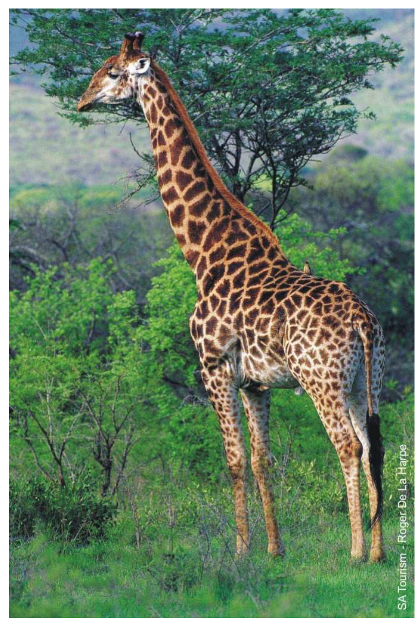


Figure 1.4: Beautiful Giraffe

This wrapfig is the only one you must be careful where to place it, it is your responsibility if it is outside of the margins

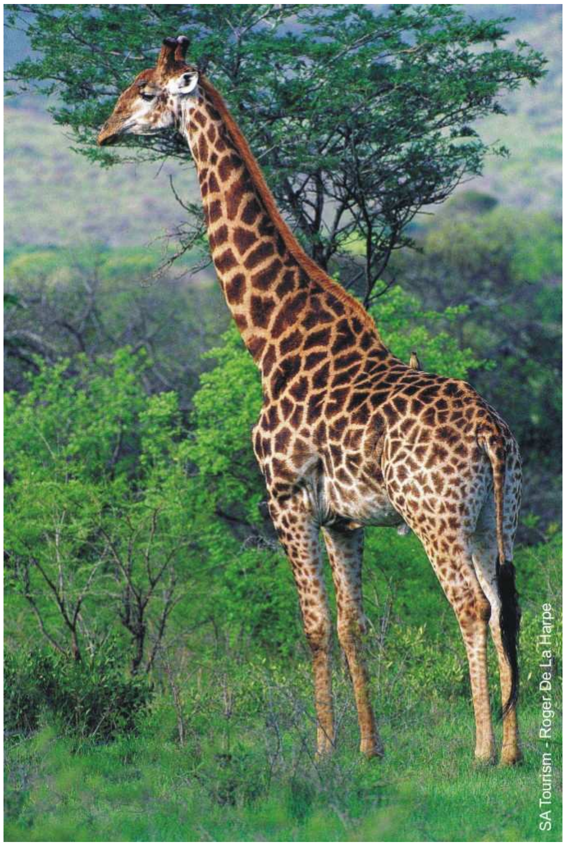


Figure 1.5: The giraffe (*Giraffa camelopardalis*) is an African even-toed ungulate mammal, the tallest of all land-living animal species. Males can be 4.8 to 5.5 metres tall and weigh up to 1,360 kilograms. The record-sized bull was 5.87 m tall and weighed approximately 2,000 kg. Females are generally slightly shorter and weigh less than the males do.

Gulls the larger species in particular are resourceful and highly-intelligent birds, demonstrating complex methods of communication and a highly-developed social structure. Certain species (e.g. the Herring Gull) have exhibited tool use behaviour. Many species of gull have learned to co-exist successfully with man and have thrived in human habitats. Others rely on kleptoparasitism to get their food.

(a) A gull **(b)** A tiger **(c)** A tiger

Figure 1.6: Pictures of animals

Here blah blah blah. Then of course more blah blah blah. Finally yeah yeah yeah. Here blah blah blah. Then of course more blah blah blah. Finally yeah yeah yeah. Here blah blah blah. Then of course more blah blah blah. Finally yeah yeah yeah. Here blah blah blah. Then of course more blah blah blah. Finally yeah yeah yeah. Here blah blah blah. Then of course more blah blah blah. Finally yeah yeah yeah. Here blah blah blah. Then of course more blah blah blah. Finally yeah yeah yeah. Here blah blah blah. Then of course more blah blah blah. Finally yeah yeah yeah. Here blah blah blah. Then of course more blah blah blah. Finally yeah yeah yeah. Here blah blah blah. Then of course more blah blah blah. Finally yeah yeah yeah.

Here blah blah blah. Then of course more blah blah blah.

Chapter 2

Rational Points in Polygons

2.1 The best female singer

In order to find a rational point on a polygon we can use one of two methods: you can choose your rational point and then find three points all rational distance away from this point and connect them to form a polygon.

Not always but sometimes, here we define blah blah. Then of course more blah blah blah. Finally yeah yeah yeah. Not always but sometimes, here we define blah blah. Then of course more blah blah blah. Finally yeah yeah yeah. Not always but sometimes, here we define blah blah.

Figure 2.1: Shakira performing *Waka Waka*, prior to World Cup in South Africa

2.2 Rational Triangles

In order to find a rational point on a polygon we can use one of two methods: you can choose your rational point and then find three points all rational distance away from this point and connect them to form a polygon. In this case we can build any polygon with the center as a rational point but the only question that arises is do we want all the lengths of its sides to be also rational? If this is the case then we should use the Rational triangles to form these polygons with rational points.

Definition 2.2.1 *A triangle is called rational if its sides and area are expressed by rational numbers.*

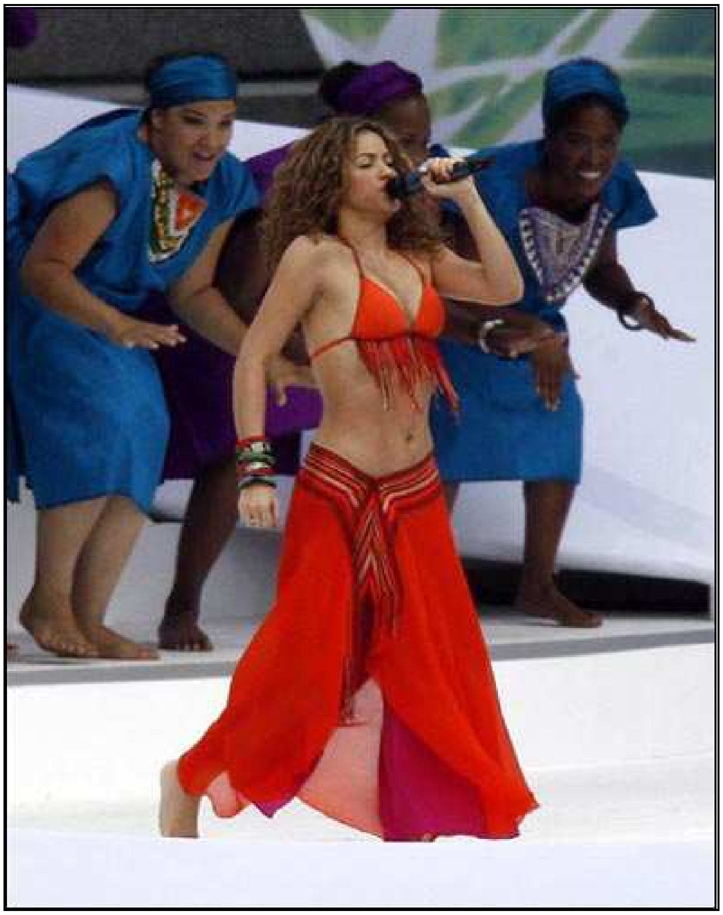
This property makes the search obvious. If we have a rational triangle, then its vertices are rational points, since the distances from that vertex to the other two vertices are rational by definition. The other obvious rational point is the vertex of the right angle formed by the base and the height of the triangle. In this section examples will be provided showing the existence of rational points on certain polygons. For most of the polygons we will show the existence of rational points on the vertices, the center, or on its sides.

2.2.1 Triangles

If we want to show examples of triangles with rational points, we should start with the simplest one: a right triangle. If we follow the pythagorean theorem we can use the first primitive pythagorean theorem, this is what Pythagoras said:

> I declare that the right triangles are the coolest of the coolest. Then of course more blah blah blah. Finally yeah yeah yeah. Here blah blah blah. Then of course more blah blah blah. Finally yeah yeah yeah. Here blah blah blah. Then of course more blah blah blah. Finally yeah yeah yeah. Here blah blah blah. Then of course more blah blah blah. Finally yeah yeah yeah. Here blah blah blah. Then of course more blah blah blah. Finally yeah yeah yeah.Here blah blah blah. Then of course more blah blah blah. Finally yeah yeah yeah.

He was of course heart-broken because his girlfriend dumped him. Here blah blah blah. Then of course more blah blah blah. Finally yeah yeah yeah. Here blah blah blah. Then of course more blah blah blah. Finally yeah yeah yeah. Here blah blah blah. Then of course more blah blah blah. Finally yeah yeah yeah. Here blah blah blah. Then of course more blah blah blah. Finally yeah yeah yeah. Here blah blah blah. Then of course more blah blah blah. Finally yeah yeah yeah.Here blah blah blah. Then of course more blah blah blah. Finally yeah yeah yeah. Finally yeah yeah yeah.



Definition 2.2.2 *A triangle is called rational if its sides and area are expressed by rational numbers.*

This property makes the search obvious. If we have a rational triangle, then its vertices are rational points, since the distances from that vertex to the other two vertices are rational by definition. The other obvious rational point is the vertex of the right angle formed by the base and the height of the triangle. In this section examples will be provided showing the existence of rational points on certain polygons. For most of the polygons we will show the existence of rational points on the vertices, the center, or on its sides.

CHAPTER 3

Rational Points on the Unit Square

3.1 Rational Points on the Edges

Given that the strategy of finding rational points inside the unit square is not easy, we change our attention to rational points on the edges. We shall then start our search for Rational Points from the outside in, thus starting on the edges. During all this chapter we will be using assuming the following: Condition: There is a rational point on the edges of the unit square with coordinates (x, y).

Corollary 3.1.1 *There exists two Pythagorean triples:* (a, b, c_1) *and* $(a, (a-b), c_2)$.

Proof. By Lemma 1.0.2 the coordinates of the rational point (x, y) are both rational, multiplying by a large number we can assume that (x, y) are integers and the unit square is now a big square of length a and let c_1 and c_2 be the distances from two vertices to the point. $a, b, (a-b), c_1, c_2$ are integers, and they remain Pythagorean triples.

CHAPTER 4

CONCLUSIONS

My Conclusion is that long time ago, it was. Then of course more blah blah blah. Finally yeah yeah yeah. My Conclusion is that long time ago, it was. Then of course more blah blah blah. Finally yeah yeah yeah. My Conclusion is that long time ago, it was. Then of course more blah blah blah. Finally yeah yeah yeah. My Conclusion is that long time ago, it was. Then of course more blah blah blah. Finally yeah yeah yeah. My Conclusion is that long time ago, it was. Then of course more blah blah blah. Finally yeah yeah yeah.

Glossary

OK-BB : Other Kilogram of Big Bow. Employed mainly for examples. Not always but sometimes, here we define blah blah.

CONAPO : Consejo Nacional de Población

RANGO : The brother of Rambo, usually when referring to fights among them.

AMIGO : Term employed by your friend who wants you to invite him a beer. Not always but sometimes, here we define blah blah. Then of course more blah blah blah. Finally yeah yeah yeah.

NASA : Not Always Scientifically Accurate. Not always but sometimes, here we define blah blah. Then of course more blah blah blah. Finally yeah yeah yeah.

SSA : Secretaría de Salubridad y Asistencia; actualmente Secretaría de Salud

OK-BB : Other Kilogram of Big Bow. Employed mainly for examples.

CONAPO : Consejo Nacional de Población. Not always but sometimes, here we define blah blah. Then of course more blah blah blah. Finally yeah yeah yeah.

RANGO : The brother of Rambo, usually when referring to fights among them.

AMIGO : Term employed by your friend who wants you to invite him a beer.

NASA : Not Always Scientifically Accurate. Not always but sometimes, here we define blah blah. Then of course more blah blah blah. Finally yeah yeah yeah.

AMIGO : Term employed by your friend who wants you to invite him a beer.

NASA : Not Always Scientifically Accurate. Not always but sometimes, here we define blah blah. Then of course more blah blah blah. Finally yeah yeah yeah.

List of Figures

(You really need to replace this single-one-page-file Here are the)

INSTRUCTIONS LICENSE INSTRUCTIONS

1. Go and open the file named: DistLicenseBeforeScanning.tex
2. Make the appropriate substitutions in the file (instructions are also found inside the file).
3. Compile that file with Latex, then dvitops, then pspdf
4. Print it
5. Sign it, Scan it, Save it as License.pdf in this folder
6. When you save it, the computer will ask you if you want to replace it with a file that already exists, say yes!