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2 This  
page  
is  
here  
to  
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page  
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bers  
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cor-  
rectly.

Do  
not  
print  
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*My*

*The-*

*sis*

*Ti-*

*tle*

A  
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the  
De-  
gree

Masters  
of  
Sci-  
ence

by

Shawn  
Richard-  
son

April,  
2014

©  
2014  
Shawn  
Richard-  
son  
ALL  
RIGHTS

RE-  
SERVED

*Signature*  
*page*  
*for*  
*the*  
*Mas-*  
*ters*  
*in*  
*Math-*  
*e-*  
*mat-*  
*ics*  
*The-*  
*sis*  
*of*

APPROVED FOR THE MATHEMATICS PROGRAM

~~Thesis Advisor's name, Thesis Advisor Date~~

~~Committee Member's name, Thesis Committee Date~~

APPROVED FOR THE UNIVERSITY

~~Dr. Gary A. Berg, AVP Extended University Date~~

To  
my  
par-  
ents,  
name  
and  
name,  
in  
grat-  
i-  
tude  
for  
their  
en-  
cour-  
age-  
ment  
and  
sup-  
port.

## Acknowledgements

## **Abstract**

My  
The-  
sis  
Ti-  
tle  
by  
Shawn  
Richard-  
son

Type  
in  
your  
ab-  
stract  
here.

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## 1. INTRODUCTION

In  
an  
at-  
tempt  
to  
match  
fin-  
ger-  
prints  
sam-  
pled  
at  
dif-  
fer-  
ent  
res-  
o-  
lu-  
tions  
and  
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dif-  
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These  
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pared  
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the  
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gles  
be-  
tween  
the  
points  
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gard  
to  
the  
length  
of  
the  
sides.  
This  
was  
done  
to  
ab-  
stract  
the  
fin-  
ger-  
prints

from  
the  
res-  
o-  
lu-  
tion  
of  
the  
orig-  
i-  
nal  
im-  
age  
as  
well  
as  
the  
an-  
gle  
at  
which  
the  
print  
was  
scanned.

## 2. RELATED

WORK

## 3. METHODOLOGY

### 3.1. Initial

**Hy-  
poth-  
e-  
sis.**

After  
extract-  
ing  
the  
co-  
or-  
di-  
nates  
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the  
minu-  
tia  
points,  
a  
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ated  
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unique  
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minu-  
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Each  
of  
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tri-  
an-  
gles  
can  
be  
stored  
in  
a  
data-  
base  
as  
a  
pair  
of  
num-  
bers,  
one  
for  
each  
of  
the  
 $2^2$   
largest  
an-  
gles.  
The  
third  
an-  
gle  
is  
not  
re-  
quired  
as  
the  
sum  
of  
the  
an-  
gles  
must

equal  
180  
de-  
grees.  
By  
stor-  
ing  
only  
the  
two  
an-  
gles,  
res-  
o-  
lu-  
tion,  
ori-  
en-  
ta-  
tion,  
and  
in-  
ver-  
sion  
of  
the  
im-  
age  
are  
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stracted.  
Fin-  
ger-  
prints  
could  
then  
be  
com-  
pared  
by  
the

num-  
ber  
of  
tri-  
an-  
gles  
in  
each  
im-  
age  
that  
match.

#### 4. RESULTS

800  
fin-  
ger-  
prints  
in  
data-  
base.  
17639  
end  
points  
and  
13015  
bi-  
fur-  
ca-  
tion  
points.

## 5. DISCUSSION

AND

FU-

TURE

WORK

5.1. Notation  
and  
Ter-  
mi-  
nol-  
ogy.  
(As  
needed...)

## 6. HYPOTHESIS

### 6.1. Initial

**Hy-**  
**poth-**  
**e-**  
**sis**  
**Re-**  
**sults.**

Us-  
ing  
the  
[In-  
sert  
Data-  
base  
Name]  
fin-  
ger-  
print  
data-  
base,  
ex-  
per-  
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men-  
tal  
re-  
sults  
show  
that  
fin-  
ger-  
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im-  
ages  
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more  
minu-  
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sig-  
nif-  
i-  
cantly  
higher  
than  
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with  
fewer  
minu-  
tia  
points,  
even  
if  
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ages  
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of  
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fin-  
ger-  
print.  
[Add  
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sult  
data  
and  
method-  
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ogy]

## 6.2. Revision

1.  
In  
or-  
der  
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tives,  
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eral  
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sions  
were  
made.  
The  
length  
of  
the  
longest  
side  
was  
added.

### 6.3. Revision

**2.**

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ing  
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largest  
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gle  
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a  
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ing  
point,  
the  
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each  
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tia  
points.

## 7. DATA

USED

FOR

ANAL-

Y-

SIS

### 7.1. FVC2002:

**Fin-  
ger-  
print  
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tion.**

Of  
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sion.  
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trol  
the  
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ity  
of  
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im-  
ages.

?,

]]maio<sub>fvc</sub>2002 :?

???

Of  
the  
120  
fin-  
gers,  
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ages  
of  
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est  
qual-  
ity  
fin-  
gers  
were  
dis-  
carded.

Of  
the  
re-  
main-  
ing

110  
fin-  
gers,  
100  
were  
des-  
ig-  
nated  
as  
set  
A  
and  
10  
as  
set  
B,  
with  
8  
of  
the  
12  
sam-  
pled  
im-  
pres-  
sions  
in-  
cluded  
?,

]]maio<sub>f</sub>vc2002 :?

???.Myresearchutilizedthe800imagesmadeavailableassetAofDB1.

## 8. MINUTIAE

EX-

TRAC-

TION

### 8.1. Image Prepa-

ra-  
tion.

The  
im-  
ages  
used  
are  
op-  
ti-  
cal  
scans  
of  
fin-  
gers  
at  
500  
dots  
per  
inch  
in  
8  
bit  
gray  
scale  
ash  
shown  
in  
Fig-  
ure  
??.

Utilizing

the  
DIP-  
im-  
age  
tool-  
box  
within  
MAT-  
LAB,  
the  
im-  
ages  
were  
en-  
hanced  
to  
re-  
duce  
mis-  
clas-  
si-  
fi-  
ca-  
tion  
of  
ridge  
end-  
ings  
and  
bifrica-  
tion  
points.  
Each  
fin-  
ger-  
print  
im-  
age  
was  
di-  
vided

into  
blocks  
of  
 $16 \times$   
16  
pix-  
els  
for  
which  
the  
vari-  
ance  
of  
each  
block  
was  
com-  
puted.  
The  
vari-  
ance  
was  
then  
com-  
pared  
with  
the  
thresh-  
old  
value.  
If  
the  
vari-  
ance  
of  
a  
block  
was  
less  
than  
the

thresh-  
old  
value,  
then  
the  
block  
was  
deleted  
from  
the  
orig-  
i-  
nal  
fig-  
ure.  
This  
pro-  
cess  
was  
car-  
ried  
out  
for  
the  
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tire  
im-  
age.  
The  
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sult-  
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was  
then  
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get

the  
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vari-  
ance  
of  
the  
given  
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age.  
The  
blocks  
were  
then  
fil-  
tered  
along  
the  
di-  
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tion  
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the  
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tion  
an-  
gle  
us-  
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value  
of  
the  
fre-  
quency  
ob-  
tained  
for  
each

block.  
A  
bi-  
nary  
mask  
was  
used  
to  
re-  
move  
points  
around  
the  
edges  
as  
well  
as  
in-  
ter-  
nal  
points  
with  
a  
low  
re-  
li-  
a-  
bil-  
ity  
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as  
gen-  
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by  
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Fil-  
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ing  
was  
done  
for  
each  
block  
us-  
ing  
the  
lo-  
cal  
ori-  
en-  
ta-  
tion  
an-  
gle  
and  
fre-  
quency.

## 8.2. Skeletonization.

A  
bi-  
nary  
im-  
age  
was  
gen-  
er-  
ated  
from  
the

age

pre-  
pared  
im-  
age  
as  
shown  
in  
Fig-  
ure  
??.  
The  
bi-  
nary  
im-  
age  
was  
then  
skele-  
tonized  
by  
re-  
mov-  
ing  
the  
outer  
layer

of  
pix-  
els  
un-  
til  
a  
sin-  
gle  
pixel  
wide  
line  
re-  
mained.

End  
pix-  
els  
were  
then  
iden-  
ti-  
fied  
as  
those  
pix-  
els  
with  
ex-  
actly  
pixel  
within  
the  
8  
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ble  
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round-  
ing  
pix-  
els.  
A

mask  
was  
ap-  
plied  
to  
these  
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sults  
to  
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i-  
nate  
the  
ar-  
eas  
around  
the  
edge  
which  
would  
oth-  
er-  
wise  
gen-  
er-  
ate  
false  
end  
points,  
as  
well  
as  
ar-  
eas  
of  
low  
con-  
fi-  
dence  
where  
the

age

im-  
age  
was  
not  
suf-  
fi-  
ciently  
clear.

### 8.3. **Identify Minu- tiae.**

Each  
of  
the  
end  
points  
re-  
main-  
ing  
af-  
ter  
the  
mask-  
ing  
pro-  
cess

was  
de-  
ter-  
mined  
to  
be  
an  
end  
point.

#### 8.4. **Minutiae Prop- er- ties.**

### 9. ANALYSIS

#### 9.1. **Triangles.**

#### 9.2. **Principal Com- po- nent Anal- y- sis.**

#### 9.3. **Matching.**

## 10. STUFF

TO

IN-

CLUDE

### 10.1. **Fingerprint Match-**

**ing.**

Re-

li-

ably

match-

ing

im-

ages

of

fin-

ger-

prints

is

ex-

tremely

dif-

fi-

cult,

mainly

due

to

large

vari-

a-

tions

in

im-

ages

of

the

same

finger.  
The  
main  
factors  
responsible  
for  
these  
large  
intra-class  
variations  
are:  
displacement,  
rotation,  
partial  
overlap,  
non-linear  
distortion,  
variable  
pressure,  
changing  
skin

con-  
di-  
tions,  
noise,  
and  
fea-  
ture  
ex-  
trac-  
tion  
errors?,  
]31]maltoni<sub>h</sub>*andbook*<sub>2</sub>003.

## 10.2. **Data Col- lected.**

For  
each  
im-  
age  
pro-  
cessed,  
the  
minu-  
tia  
points  
are  
de-  
ter-  
mined.  
Then  
for  
each  
minu-  
tia  
point,  
the  
lo-  
ca-  
tion,

direction,  
and  
type  
are  
determined.  
The  
location  
is  
extracted  
as  
the  
*xy*-  
coordinate  
in  
the  
image  
and  
the  
direction  
is  
the  
angle  
relative  
to  
a  
line  
running

from  
the  
ori-  
gin,  
in-  
creas-  
ing  
along  
the  
 $x$   
axis.  
The  
type  
is  
clas-  
si-  
fied  
as  
ei-  
ther  
an  
end  
or  
bi-  
fur-  
ca-  
tion  
point.

### 10.3. **Triangulation.**

The  
minu-  
tia  
are  
then  
grouped  
into  
sets  
of  
3,

forming  
a  
triangle.  
Using  
the  
 $xy$ -  
coordinates  
of  
each  
point,  
the  
angles  
and  
lengths  
of  
each  
side  
can  
be  
cal-  
cu-  
lated.  
For  
each  
tri-  
an-  
gle,  
the  
largest  
an-  
gle  
was  
first  
de-  
ter-  
mined,

then  
tak-  
ing  
a  
clock-  
wise  
ro-  
ta-  
tion,  
the  
sec-  
ond  
an-  
gle  
is  
de-  
ter-  
mined.  
Along  
with  
the  
dis-  
tance  
be-  
tween  
minu-  
tia  
at  
the  
ve-  
rac-  
i-  
ties  
of  
these  
two  
an-  
gles,  
the

tri-  
an-  
gle  
can  
be  
be  
fully  
rep-  
re-  
sented.  
Hypnosis:  
Us-  
ing  
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cen-  
ter  
point  
of  
the  
tri-  
an-  
gle  
as  
a  
ref-  
er-  
ence  
would  
min-  
i-  
mize  
the  
im-  
pact  
of  
er-  
rors  
caused  
ei-  
ther  
by

dis-  
tor-  
tion  
or  
er-  
rors  
in  
the  
ex-  
trac-  
tion  
pro-  
cess.  
The  
di-  
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tion  
of  
the  
minu-  
tia  
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lated  
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of  
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rors  
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the  
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ca-  
tions  
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minu-  
tia,  
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cess.

*Proof.*

Let

$A, B$

and

$C$

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trary

minu-

tia

points.

Let

$a$

be

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$B$

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$C$ ,

then

$a =$

$$\sqrt{(B_x - C_x)^2 + (B_y - C_y)^2}$$

$b =$

$$\sqrt{(A_x - C_x)^2 + (A_y - C_y)^2}$$

$c =$

$$\sqrt{(A_x - B_x)^2 + (A_y - B_y)^2}$$

□

$\theta =$

$$\arctan\left(\frac{\textit{opposite}}{\textit{adjacent}}\right) =$$

$$\arctan\left(\frac{\Delta x}{\Delta y}\right)$$

$\cot \theta =$

$$\left(\frac{\textit{adjacent}}{\textit{opposite}}\right) =$$

$$\left(\frac{\Delta y}{\Delta x}\right)$$

If

$$\Delta y <$$

0,

$\theta =$

$\theta +$   
 $\pi$   
If  
 $\Delta x =$   
 $0, \theta =$   
 $\frac{\pi}{2}$

## 10.4. Training

### **Data.**

The data-base used consists of 8 different images of 100 unique fingers for a total of 800 fingerprint images. One image of finger was chosen,

using the random function within MySQL, and reserved for the testing set. The remaining 700 images were used as a training set. Statistics for the training data are shown in

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Component	Minimum	Maximum	Average	$\sigma$
Angle $\alpha$	1.0483928950	3.1410122706	1.7797521977	0.4721377805
Angle $\beta$	0.0001109755	1.5455915815	0.6718969221	0.3484298449
Direction a	-3.1415926535	3.1415926536	0.0167706757	1.8339501956
Direction b	-3.1415926535	3.1415926536	-0.0314165278	1.8484983038
Direction c	-3.1415926535	3.1411944932	-0.0277245946	1.8392294858
Length	2.2360679775	335.6262802583	58.0791589147	31.5340026405

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## 10.5. Principle

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